

Making Two Wires Square to Each Other

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A Fast Method for 3-4-5 Triangles

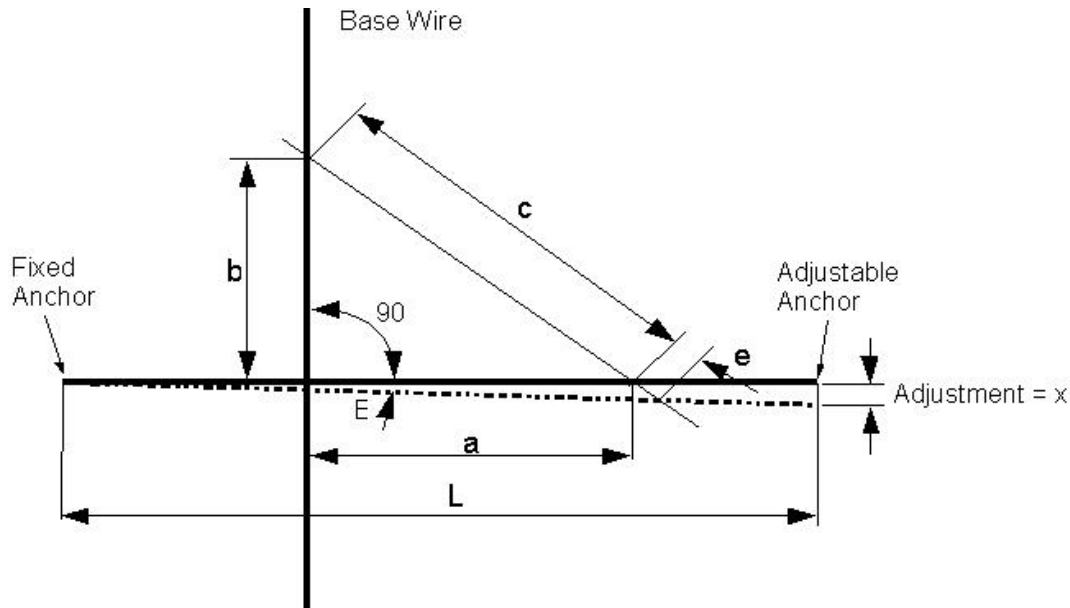
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A common method of setting two wires or strings square to each other is to use the perfect square of a 3-4-5 triangle. This method can be surprisingly accurate, however, it takes some patience to adjust the ends of the wires so the measurement of the hypotenuse is exactly "5". Generally, a lot of trial-and-error adjustments to one end of the wire.

There is a formula for determining the distance that one end of the wire needs to be adjusted if you know the error of hypotenuse and the length of the wire. The formula works for any triangle, not just a 3-4-5 triangle, but if you use the 3-4-5 triangle, the calculation for the adjustment is simplified.

The diagram below shows the layout of the two wires and the measuring triangle. The hypotenuse is

$$c = \sqrt{a^2 + b^2}$$



- Step 1. Mark 'a' and 'b' on the two wires.
- Step 2. Measure the length 'L' of the wire that will be adjusted.
- Step 3. Measure the hypotenuse. There will be an error 'e' from the ideal number.
- Step 4. Calculate the distance to move one end of the wire:

$$x = \frac{Lce}{ab}$$

If you used a 3-4-5 triangle, then this formula reduces to:

$$x = (2.083) \left(\frac{Le}{c} \right) \text{ which is close enough to } \frac{2Le}{c} \text{ for most practical purposes.}$$

Example: 45° Triangle

$$\begin{aligned}L &= 200'' \\ a &= 100''; \quad b = 100'' \\ c &= 141.421'' \text{ (by calculation)} \\ e &= 1/16'' (= 0.062'')\end{aligned}$$

$$\text{Solution: } x = \frac{((200)(141.421)(0.062))}{((100)(100))} = 0.175 \text{ (which is just short of } 3/16'' = 0.188'')$$

Example: 3-4-5 Triangle

$$\begin{aligned}L &= 200'' \\ a &= 100''; \quad b = 75'' \\ c &= 125'' \text{ (by calculation)} \\ e &= 1/16'' (= 0.062'')\end{aligned}$$

$$\text{Solution: } x = \frac{(2)(0.062)}{(125)} = 0.100 \text{ (which is just over } 3/32'' = 0.094'')$$

Proof:

$$\begin{aligned}\text{Law of Cosines for a triangle: } 2ab\cos(90+E) &= a^2 + b^2 - m^2 = c^2 - m^2 \\ \text{where } m &= c+e \text{ (the measured hypotenuse)}\end{aligned}$$

Using:

$$\begin{aligned}\cos(90+E) &= -\sin(E) \\ c^2 - m^2 &= c^2 - (c+e)^2 = -2ce - e^2\end{aligned}$$

For $c \gg e$

$$2ce + e^2 \approx 2ce$$

$$\sin(E) \approx \frac{x}{L}$$

Then:

$$x = \frac{Lce}{ab}$$

For the special case of a 3-4-5 triangle:

$$x = \left(\frac{L}{c}\right)\left(\frac{c}{a}\right)\left(\frac{c}{b}\right)e = \left(\frac{5}{3}\right)\left(\frac{5}{4}\right)\left(\frac{L}{c}\right)e = \left(\frac{25}{12}\right)\left(\frac{L}{c}\right)e = 2.083\left(\frac{L}{c}\right)e$$

